Regional Mathematical Olympiad

Time: 3 hours **October 3, 2025**

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Show that for all positive integers k, there exists a positive integer n such that $n \cdot 2^k - 7$ is a perfect square.

2. Abel and BannabelOffense are playing a game. On a $4n \times 4n$ grid of squares, BannabelOffense places exactly $4n$ tokens such that each row and each column contains one token. In a step, Abel can move a token horizontally or vertically to a neighboring square. Several tokens may occupy the same square at the same time.

   If Abel can move the tokens so that they all lie on a single main diagonal of the grid in no more than $6n^2$ moves, Abel wins. However, if Abel is not able to do so, then BannabelOffense will ban Abel from Sophie. Show that Abel can always avoid getting banned, for any configuration of tokens.

   A main diagonal of the grid is a diagonal that contains $4n$ squares. Note that there are two main diagonals.

3. Let ABC be a triangle with $\angle A < \angle B < 90^\circ$ and let Γ be the circumcircle of $\triangle ABC$. The tangents to Γ at A and C meet at P. The lines AB and PC intersect at Q. It is given that $[\triangle ACP] = [\triangle ABC] = [\triangle BQC]$. Find, with proof, the measure of angle $\angle BCA$.

   Note: $[\triangle XYZ]$ denotes the area of triangle $\triangle XYZ$.

4. Find whether the following inequality holds for all non-negative real numbers x, y, z with $x \geq y$:
$$\frac{x^3 - y^3 + z^3 + 1}{6} \geq (x - y)\sqrt{xyz}$$

5. Two positive integers a and b are said to be prime-related if $a = pb$ or $b = pa$ for some prime p. A positive integer n is dividocray if it has at least three divisors, and all the divisors can be arranged without repetition in a circle so that any two adjacent divisors are prime-related. Find all dividocray numbers.

6. For a sequence $\{a_1, a_2, a_3, \dots\}$ of real numbers it is known that for all $n \geq 2$, $a_n = a_{n-1} + a_{n+2}$. What is the largest number of consecutive elements in this sequence that can all be positive?

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